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# 5 : b, c, e

For the following interpretation (D = {a, b})

|  |  |  |  |
| --- | --- | --- | --- |
| P(a,a) | P(a,b) | P(b,a) | P(b,b) |
| T | F | F | T |

Determine the truth value of the following formulas (quantified propositions)

b. ∀x∀y P(x, y)

x = a, y = a: = T

x = a, y = b: = F 🡺 One is false therefore **∀x∀y P(x, y) = F**

c. ∃x ∀y P(x, y)

x = a, y = a: = T

x = a, y = b: = F 🡺 **One is false therefore ∃x ∀y P(x, y) = F**

e. ∀x ∀y (P(x,y) 🡪 P(y,x))

x = a, y = a: = T : T 🡺 T = T

x = a, y = b: = F: F 🡺 F = T

x = b, y = a: = F: F 🡺 F = T

x = b, y = b: = T: T 🡺 T = T

**All are T therefore ∀x ∀y (P(x,y) 🡺 P(y,x)) = T**

7. Consider the following interpretation:

Domain : D = {1 ,2 }

Assignment for function f:

|  |  |
| --- | --- |
| f(1) | f(2) |
| 2 | 1 |

Assignment for predicate P:

|  |  |  |  |
| --- | --- | --- | --- |
| P(1,1) | P(1,2) | P(2,1) | P(2,2) |
| T | T | F | F |

Evaluate the truth value of the following formulas in the above interpretation:

1. P(a, f(a)) ∧ P(b, f(b))

**a = 1, b = 1: P(1, f(1)) ∧ P(1, f(1)): T ∧ T = T**

a = 1, b = 2: **P(1, f(1)) ∧ P(2, f(2)): T ∧ F = F**

**🡺 One is false therefore P(a, f(a)) ∧ P(b, f(b)) = F**

1. (∀x) (∃y) P(y, x)

Y = 1, x = 1: = T 🗸

Y = 2, x = 1: = F

Y = 1, x = 2: = T 🗸

Y = 2, x = 2: = F

1. Both x’s have a y to make **(∀x) (∃y) P(y, x) therefore the truth value of (∀x) (∃y) P(y, x) = T**